M. A. Latcha
Assistant Professor.
Department of Mechanical Engineering,
Oakland University,
Rochester, MI 48309

A. Akay
Professor.
Department of Mechanical Engineering,
Wayne State University,
Detroit, MI 48202

Effects of Viscous and Thermal Losses on Radiation from an Infinite Plate

This paper is concerned with the behavior of sound radiation from bending waves of an infinite plate in a fluid with viscous and thermal losses. The radiation field variables are obtained through a solution of the equation of motion for viscothermal fluids. Acoustic energy flux in the fluid is investigated, with particular emphasis on its behavior near the plate surface. Energy flux is shown to change direction in a region near the plate. The motion of the particles orbiting in elliptical patterns also exhibits a similar change of direction as a result of viscous and thermal losses. A notable effect of the viscothermal losses is the radiation of acoustic power from the plate below the critical frequency.

1 Introduction

It is well known that, in ideal fluids, radiation of sound from bending waves of an infinite plate is confined to frequencies above the critical frequency, i.e., the frequency at which the speed of a free-running bending wave is the same as the sound speed in the surrounding fluid. Below the critical frequency, pressure distribution resulting from the bending wave vibrations decays exponentially away from the plate surface, and, under steady-state conditions, there is no net energy exchange between the plate and the fluid. An exception to this was shown to be due to viscothermal losses in the fluid. Ingard and Akay (1987) showed that losses in the viscothermal boundary layer lead to radiation from bending waves below the critical frequency.

The purpose of this paper is to extend the analysis of Ingard and Akay (1987) on boundary layer effects, by considering the first-order viscothermal losses in the entire fluid. The results show that below the critical frequency there is indeed a transfer of power into the fluid, but its propagation is predominantly confined to the boundary layer. The effects of viscothermal losses on the direction of acoustic energy flux is also noteworthy. The energy flux starts in a direction normal to the plate. At a distance of approximately five times the boundary layer thickness, it reaches the same direction as that in a loss-free medium.

2 Analysis

When the viscosity and heat conduction in a fluid medium are not negligible, the acoustic behavior of the fluid medium can no longer be described by the simple Helmholtz equation. The dynamic properties of lossy fluids are governed by a trio of coupled differential equations that equations describe three distinct waves in the fluid. The first of these is a propagating wave, closely related to the pressure waves in a loss-free fluid, attenuated by viscothermal effects in the bulk of the fluid. The other two, the thermal and viscous waves, are diffusive in nature and are most important near the fluid-solid boundaries. Detailed derivations and the physics of the equations that govern the motion of lossy fluids can be found in Morse and Ingard (1986) and Temkin (1981). Following the notation in Morse and Ingard (1986), the governing equations are written as

$$\nabla^2\phi_p + k_p^2\phi_p = 0 \tag{1a}$$

$$\nabla^2\phi_t + k_t^2\phi_t = 0 \tag{1b}$$

$$\nabla^2\Omega + k_v^2\Omega = 0 \tag{1c}$$

where $\phi_p + \phi_t$ is the velocity potential and Ω is the vector potential. The wavenumbers, k_p, k_t, and k_v, for the propagating, thermal, and transverse velocity waves are expressed as

$$k_p^2 = k^2\ \{1 + ik[l_v' + (\gamma - 1)l_h]\}$$

$$k_t^2 = ik/l_h$$

$$k_v^2 = ik/l_v$$

where $k = \omega/c$ is the acoustic wavenumber, with c being the speed of sound in the fluid. The terms l_h, l_v, and l_v' are characteristic lengths in the fluid based on thermal conduction and viscous properties of the fluid, and are defined as (Morse and Ingard, 1986)

$$l_h = K/\rho c C_p; \quad l_v = \mu/\rho c; \quad l_v' = (4\mu/3\rho c)(1 + 3\alpha/4\mu) \tag{2}$$

where K is the thermal conduction coefficient, ρ is the density of the fluid, and C_p is the specific heat at constant pressure. μ and α are the coefficients of shear and bulk viscosities, respectively. The ratio of the specific heats is designated by γ.

The boundary conditions necessary for the solution of the differential equations are specified for the the particle velocity

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and temperature fluctuations at the plate-fluid interface. As explained by Ingard and Akay (1987), the particle velocity at the surface must be equal to the velocity of the plate surface, and the thermal fluctuations are negligibly small and are assumed to vanish at the interface of a fluid with a metal surface.

The particle velocity, defined in terms of the velocity and vector potentials, is expressed as

$$\mathbf{u} = \nabla(\phi_p + \phi_t) + \nabla \times \mathbf{\Omega} \tag{3}$$

Consider a harmonic distribution or one component of the Fourier representation of an arbitrary spatial distribution of the bending waves represented by a bending wave number k_x, oscillating at a frequency $\omega/2\pi$. Assuming that the neutral axis of the plate coincides with the $x-z$ plane, for one-dimensional waves in the x-direction, the boundary condition becomes

$$\mathbf{u}(x,h/2,\omega) = U_0 e^{i(k_x x - \omega t)}\mathbf{j}$$

where h is the thickness of the plate and U_0 is the velocity amplitude of the plate vibrations. Similarly, the temperature fluctuations are expressed as

$$\tau = (i/\beta\omega)[(1 - i\gamma k l_v')\nabla^2(\phi_p + \phi_t) + \gamma k^2(\phi_p + \phi_t)]$$

where β is the coefficient of thermal expansion. The corresponding boundary condition is

$$\tau(x,h/2,\omega) = 0$$

Then the potentials in Eqs. (1a)–(1c) are obtained for the fluid on one side of the plate as:

$$\phi_p = -i(U_0/D)e^{i[k_x x + k_y y' - \omega t]} \tag{4a}$$

$$\phi_t = -iA_t(U_0/D)e^{i[k_x x + k_\tau y' - \omega t]} \tag{4b}$$

$$\mathbf{\Omega} = i(A_t + 1)(U_0/D)(k_x/k\nu)e^{i[k_x x + k_\nu y' - \omega t]}\mathbf{k} \tag{4c}$$

where

$$y' = y - h/2; \quad D = k_y + A_t k_\tau + (A_t + 1)k_x^2/k_\nu$$

$$k_y^2 = k_p^2 - k_x^2; \quad k_\tau^2 = k_t^2 - k_x^2; \quad k_\nu^2 = k_v^2 - k_x^2$$

$$A_t = [\gamma k^2 - (1 - i\gamma k l_v')k_p^2]/[(1 - i\gamma k l_v')k_t^2 - \gamma k^2]$$
$$\approx -i(\gamma - 1)k l_h/(1 + 2\gamma k l_v')$$

A_t is the ratio of the amplitudes of the diffusive wave ϕ_t and the propagating wave ϕ_p at the plate-fluid interface. Its approximate value is obtained by neglecting the second- and higher-order terms in kl_h and kl_v'. The vector potential $\mathbf{\Omega}$ only has a $\mathbf{k}$-component, always normal to the $x-y$ plane. Similar equations for the potential can be written for the fluid on the other side of the plate by considering the propagation directions.

The pressure and particle velocity for the region $h/2 < y < \infty$ in the fluid can now be written explicitly. The pressure fluctuations in terms of the potentials are described as

$$p = i\omega\rho(1 + ikl_v')\phi_p + i\omega\rho\,[(l_h - l_v')/l_h]\phi_t$$

Substituting for the potentials results in

$$p = (\omega\rho U_0/D)\left\{(1 + ikl_v')e^{ik_y y'} + [(l_h - l_v')/l_h]\,A_t e^{ik_\tau y'}\right\}e^{i(k_x x - \omega t)} \tag{5}$$

Similarly, the particle velocity is obtained by substituting the potentials in Eq. (3)

$$\mathbf{u} = (U_0/D)(u_x\mathbf{i} + u_y\mathbf{j})e^{i(k_x x - \omega t)}$$

where

$$u_x = k_x\,[e^{ik_y y'} + A_t e^{ik_\tau y'} - (A_t + 1)e^{ik_\nu y'}] \tag{6a}$$

$$u_y = k_y e^{ik_y y'} + A_t k_\tau e^{ik_\tau y'} + (A_t + 1)(k_x^2/k_\nu)e^{ik_\nu y'} \tag{6b}$$

With these expressions the radiation impedance on the surface of a plate can be expressed as

$$z = \left.\frac{p(k_x, k_y, \omega)}{u(k_x, k_y, \omega)}\right|_{y=0} = \frac{\omega\rho}{D}(1 + i\delta) \tag{7}$$

where

$$\delta = k[l_h + \gamma(l_v' - l_h)]/(1 + 2\gamma_1 k l_v')$$

and $k = \omega/c$.

The direction of the wave propagation in the fluid is defined by the angle θ that the velocity vector makes with the plate surface, and is given by

$$\theta = \arctan\,(u_y/u_x) \tag{8}$$

where, from the expressions for the velocity components given in Eqs. (6), it is clear that the direction of propagation of the waves depends on the distance from the surface of the plate. This point will be explored further in the next section.

For the purpose of comparison, the expressions for the pressure and particle velocity in a loss-free fluid are obtained by setting the coefficients of thermal conduction and bulk and shear viscosities to zero, resulting in the well-known expressions

$$\mathbf{u} = (U_0/k_y)(k_x\mathbf{i} + k_y\mathbf{j})e^{i[k_x x + k_y y' - \omega t]} \tag{9}$$

$$p = \omega\rho(U_0/k_y)e^{i[k_x x + k_y y' - \omega t]} \tag{10}$$

where $k_y^2 = (\omega/c)^2 - k_x^2$.

Of course, in the absence of viscothermal losses, the direction of propagation defined by the ratio of the particle velocity components no longer depends on the distance, y', from the plate surface.

Energy Flux. A consequence of the viscothermal losses is attenuation of the acoustic energy flux with distance from the surface of the plate. As a result, the intensity expression is position-dependent. Nevertheless, the intensity is still defined in the usual manner:

$$\mathbf{I}(k_x,k_y,\omega) = \Re\{p(k_x,k_y,\omega)\mathbf{u}^*(k_x,k_y,\omega)\} \tag{11}$$

where the asterisk denotes the conjugate of the complex quantity and $\Re$ indicates the real part of the quantity in curly brackets.

Normalized radiation resistance, ϵ, is obtained in a similar manner to that given by Ingard and Akay (1987) by normalizing the magnitude of intensity in Eq. (11) with respect to its value for $k_x = 0$, evaluated on the surface, $y' = 0$:

$$\epsilon = \frac{I(k_x, k_y, \omega)}{I(0, k_y, \omega)|_{y'=0}} \tag{12}$$

This normalization will be used to compare the magnitude of energy flux in the medium with that from a piston-like surface and without any dissipation due to viscothermal losses.

3 Results

Aside from the attenuation of sound propagating in a medium, viscothermal losses induce radiation below the critical frequency of bending waves and cause a change in the direction of radiation of the plane waves. Such changes take place predominantly within or near the viscous and thermal boundary layers. In order to examine the behavior of sound waves near the boundary layers, we introduce a smaller-scale normalization constant than for distances in the far field. While the far field distances are normalized with respect to the bending wavelength as $k_x y'$, to normalize the distances in the boundary layer range we define an "average" boundary layer thickness as:

$$d_{av} = \frac{1}{2}(d_v + d_h) \tag{13}$$

where d_v, the viscous, and d_h the thermal boundary layer thick-

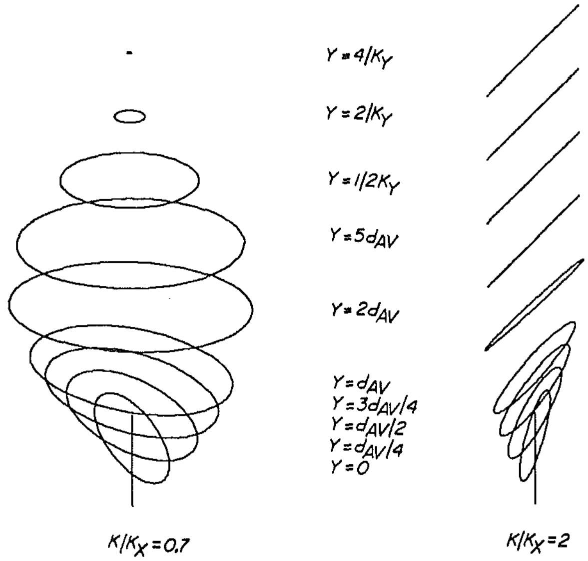

Fig. 1 Particle trajectories in a lossy medium

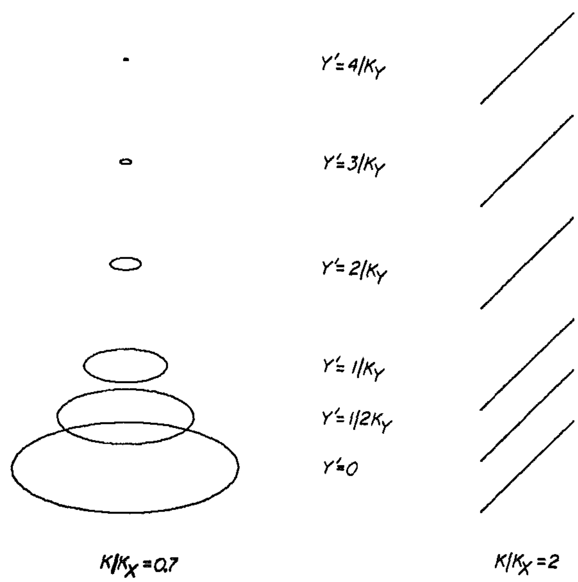

Fig. 2 Particle trajectories in a loss-free medium

nesses, are obtained from the relationships $kl_v = (kd_v)^2/2$ and $kl_h = (kd_h)^2/2$ (Morse and Ingard, 1986).

Before examining the nature of power transfer from the bending waves to the fluid medium, it is of interest to consider the influence of viscothermal losses on the behavior of particles near and away from the plate surface.

Particle Trajectories. Particle motion induced by bending waves in a loss-free fluid is an oscillation along a straight line in the direction of radiation when $k_x < k$, but describes an ellipse when $k_x > k$ (Brillouin, 1952 and Cremer, Heckl, and Ungar, 1973). Above the critical frequency, $k_x < k$, the character of the oscillations remains the same at all distances, including on the surface of the plate. Below the critical frequency, however, the axes of the ellipses shrink with increasing distance normal to the plate. The influence of viscothermal losses on particle motion is illustrated by plotting the x- and y-components of particle displacement by numerically integrating the velocity expression given in Eq. (6). The elliptical paths taken by the particles are shown in Fig. 1 and compared

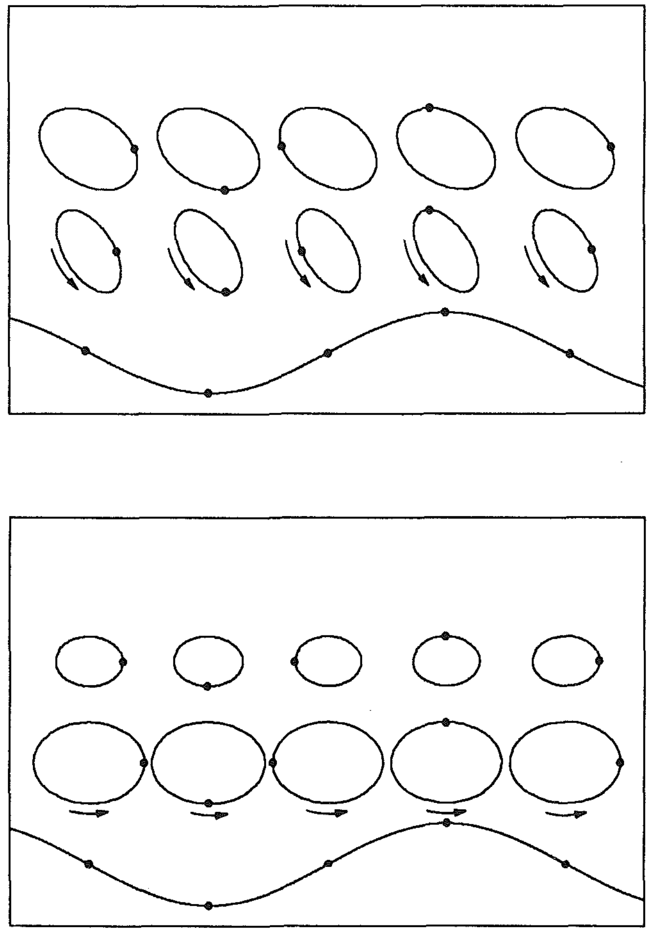

Fig. 3 The phase differences between the trajectories of the particles one quarter of a wavelength apart along the plate surface for lossy (above) and loss-free fluids

with corresponding paths in a loss-free medium, shown in Fig. 2. The main difference is that now the viscous losses do not permit slip on the surface of the plate unlike that in a loss-free fluid. Therefore, particle velocity on the plate surface is always in the normal direction. Above the critical frequency, near the plate surface the particle motion describes an ellipse. The direction of the major axis of the elliptical trajectory changes from nearly normal to the plate surface to the same direction as in a loss-free medium.The transition takes place within a distance of approximately several average boundary layer thicknesses. The minor axis of the ellipse has the largest value just off the plate, and decreases exponentially to a negligible but nonzero value within approximately five times the average boundary layer thickness. The minor axes may be interpreted to represent the losses caused by the viscothermal properties of the fluid. It is clear that such losses are predominant inside the boundary layer and are still significant for distances up to several times the boundary layer.

Below the critical frequency, again due to the viscosity of the fluid, particle velocity on the plate surface has only a normal component. The direction of the major axis of the ellipse changes from nearly normal just off the plate surface and becomes parallel to it in a distance of approximately five times the boundary layer thickness. As will be shown in the next section, below the critical frequency, power transferred from the plate to the fluid is dissipated predominantly within the distance $5d_{av}$, where the major axis of the elliptical orbit becomes parallel to the plate surface. In other words, power transfer from the bending waves into the fluid is confined to a distance of approximately $5d_{av}$ from the plate surface. This

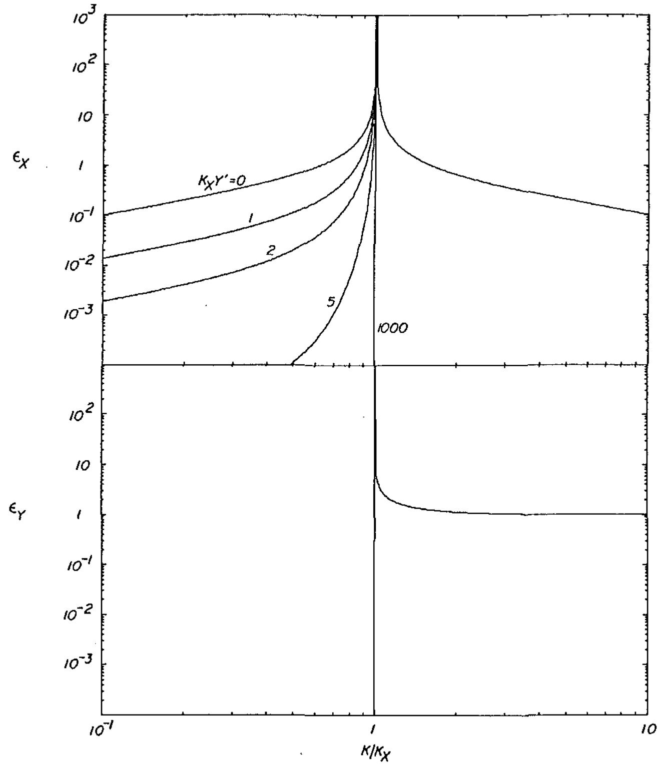

Fig. 4 The tangential (above) and normal components of the normalized intensity in a loss-free medium. Below the critical frequency, there is energy flux in the tangential direction near the plate surface.

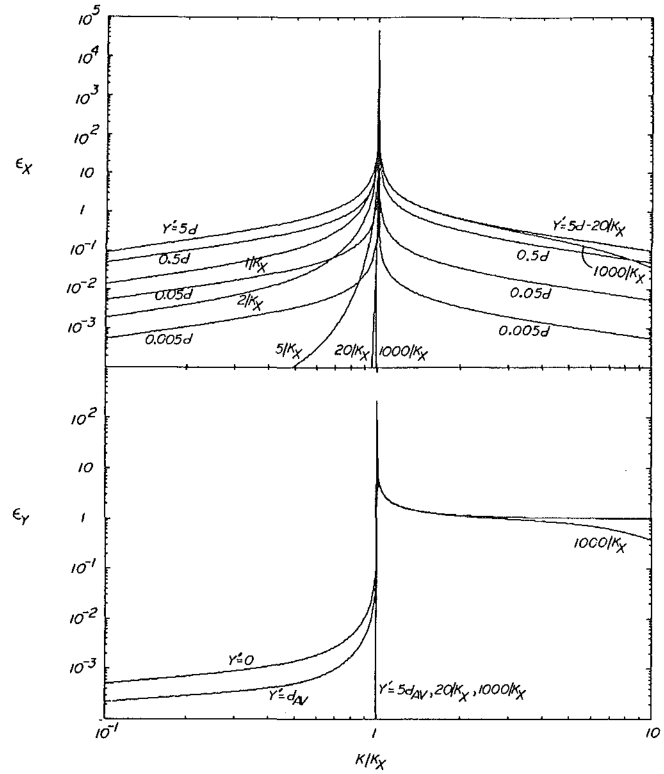

Fig. 5 The tangential (above) and normal components of the normalized intensity in a lossy fluid. In both cases, there is energy flux below the critical frequency, but it is confined to the nearfield of the plate surface.

is not the case in a loss-free fluid, where there is no net power transfer between the plate and the fluid below the critical frequency under steady-state conditions; as in the case of loss-free fluids, the motion of the fluid particles diminishes exponentially with distance from the plate.

The relative phases of particle trajectories a quarter of a wavelength apart along the plate are indicated with arrows for loss-free and lossy fluids in Fig. 3. The relationship between the particle trajectories and plate bending waves is also shown in Fig. 3, where the position of the bending wave shown in the figure corresponds to the circle on each trajectory. As points on the plate surface go through a transverse motion, the particles follow the direction indicated on the trajectories.

Energy Flux. It is known that in loss-free fluids at frequencies much higher than the critical frequency, the far field radiation from bending waves in an infinite plate is similar to that from a flat piston of the same size, and is in a direction normal to the radiating surface. This similarity is indicated by the unit value of the normalized radiation resistance at high frequencies. The coherent radiation at the critical frequency gives rise to a much higher amplitude of radiation intensity, and in a direction parallel to the surface. However, below the critical frequency, there is no radiation, and the perturbations are confined to the vicinity (nearfield) of the plate surface without a net power transfer from the bending waves to the fluid.

In the case of radiation into a fluid with viscothermal losses, above the critical frequency, radiation intensity is attenuated with distance from the plate. Below the critical frequency, however, the losses in the fluid lead to a power transfer from the bending vibrations to the fluid. The nature of this power transfer will be explored below.

Consider the components of the radiated energy flux normal and parallel to the surface. From the definitions given in Eqs. (11) and (12), substitution for the loss-free values of the pressure and particle velocity gives

$$\epsilon_x = \begin{cases} (k_1k_x)/(k_x^2-k_1^2)e^{-2\sqrt{k_x^2-k_1^2}y'} & \text{for } k_1 < k_x; \\ (k_1k_x)/(k_1^2-k_x^2) & \text{for } k_1 > k_x \end{cases} \tag{14}$$

$$\epsilon_y = \begin{cases} 0 & \text{for } k_1 < k_x; \\ k_1/\sqrt{k_1^2-k_x^2} & \text{for } k_1 > k_x \end{cases} \tag{15}$$

From the plots of Eqs. (14) and (15) in Fig. 4, it is seen that the normal component of the intensity is identical to the magnitude of intensity commonly used to describe the radiation efficiency of infinite flat plates. The parallel component of intensity exhibits the same characteristics in the far field. However, near the surface of the plate there is an energy flux parallel to the plate which decays in amplitude with increasing distance from the plate surface, as shown in Fig. 4.

The influence of viscothermal losses on radiation resistance is seen in Fig. 5. The normal component of the intensity now has a finite amplitude at the critical frequency as opposed to an infinite amplitude in loss-free fluids. At high frequency and large distance combinations radiation resistance drops off. Below the critical frequency, energy flux has a normal component but is confined to the boundary layer. It is highest on the plate surface (where the particle velocity is entirely in the normal direction) and decreases with distance away from the plate. Outside of the boundary layer it is negligible.

The parallel component of the normalized intensity, ϵ_x, below the critical frequency, on the other hand, starts to build

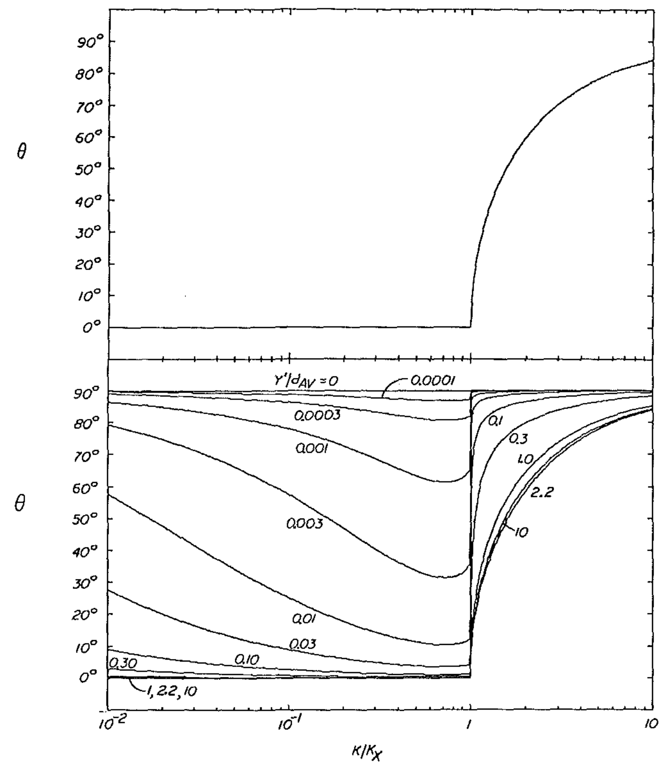

Fig. 6 Direction of radiation taken from the surface of the plate in loss-free (above) and lossy fluids. In lossy fluids, the direction of the energy flux depends on the distance from the plate.

up with distance from the plate, reaching a maximum value around $y \approx 5d_{av}$, and decreases with distance thereafter. Above the critical frequency, ϵ_x again builds up with distance from the plate and reaches a "steady-state" value within a distance of $y \approx 5d_{av}$. Its behavior at high frequency and large distance combinations is similar to the y-component of intensity.

The direction of radiation is examined by plotting the angle between the normal and parallel components of the intensity given by Eq. (8) and is shown in Fig. 6 for lossy and loss-free fluids. It is seen that at large distances normal to the surface the direction of radiation is unaffected by viscothermal losses. It is parallel to the plate below and at the critical frequency, reaching a direction normal to the plate at very high frequencies. Very close to the plate, both below and above the critical frequency, the energy flux is nearly normal to the surface. With increasing distance from the plate surface, energy flux changes its direction to a parallel configuration below the critical frequency; above the critical frequency its direction changes to that defined by θ, given above.

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